On the TT-transform and its diagonal elements

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Abstract*—The TT-transform stands for time-time transform and has been derived as an inverse Fourier transform of the time-frequency S-transform. So far, it has been proposed that the diagonal of the TT-transform can be used for signal characterisation. We show here an alternative and simplified derivation of the TT-transform which enables a better understanding of this transform.

Keywords: TT-transform, *S*-transform, time-frequency localisation, time-time analysis

I. Introduction

In disciplines such as music or geophysics, signals are non stationary. The need for processing such signals has led to the appearance of several types of time varying frequency filters, such as the short time Fourier transform [1], wavelets [2], and more recently the *S*-transform (ST) [3]. These transforms introduce redundancy passing from a 1D time signal to a 2D time-frequency (or time-scale) signal. In 2003, [4] introduced a new transform based on the ST and called the TT-transform. It includes redundancy in time passing from a 1D time signal to a 2D time-time signal. Until now, this transform has seen little application [5] and in general, interest has mainly been focused on the diagonal part. The aim of this correspondence is to show a very simple way of computing the diagonal part of the TT-transform and to give a clear interpretation of it.

In the next section, the *S*- and TT-transforms will be reviewed. Section III will demonstrate a simplified way to compute the latter. Section IV will show examples and the last section will conclude this presentation.

II. The *S*-transform and the TT-transform

The ST of a signal $u(t)$ is defined as [3]:

$$S(\tau, f) = \int_{-\infty}^{\infty} u(t) w(t-\tau, f) \mathrm{e}^{-2i\pi f t} \mathrm{d}t \qquad (1)$$

$w(t, f)$ being a 1-mean window, generally a Gaussian with a variance of *1/f* .

Starting from the ST, [4] develop the TT-transform. The concept is simply based on the inverse Fourier transform of the ST with respect to its frequency component:

$$TT(t, \tau) = \int_{-\infty}^{\infty} S(\tau, f) \mathrm{e}^{2i\pi f t} \mathrm{d}f \qquad (2)$$

Both TT and S are matrices containing redundant information, the ST being in the time-frequency domain while the TT-transform is in the time-time domain. Both transforms are easily invertible.

III. A simplified way of computing the diagonal of the TT

In the applications using the TT [5], only its diagonal part has been used as most of the information in the TT is concentrated in its diagonal terms. We will hence concentrate on this part. For details on the rest of the transform, see [6]. We have shown, [6], that

$$TT(t, t) = \mathcal{F}^{-1}\{U(f)G(f)\} \qquad (3)$$

where $\mathcal{F}$ represents the Fourier transform, $\mathcal{F}^{-1}$ its inverse, U is the Fourier transform of the signal u and $G(f) = k\pi^2|f|$, k being a constant.

This is an important result as by using this formula, not only can we completely forget about the use of the *S*- and the TT-transforms but also we can much more easily understand its behaviour. Indeed, (3) reveals that the diagonal terms of the TT-transform are just a frequency filtered version of the original signal, putting more emphasis on high frequencies.

IV. Examples of application

The first example, Fig. 1, is a combination of two successive chirps. In the bottom plot, some Gaussian noise has been added to the same signal.

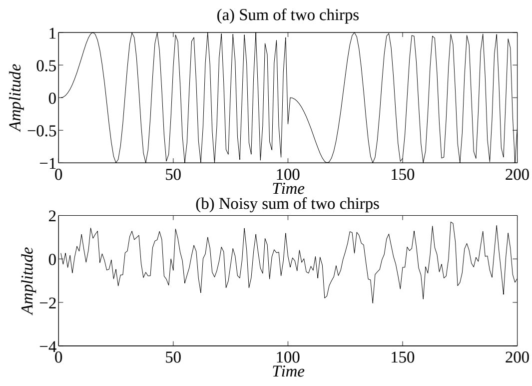
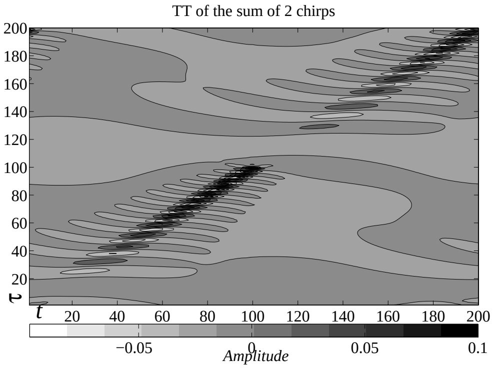


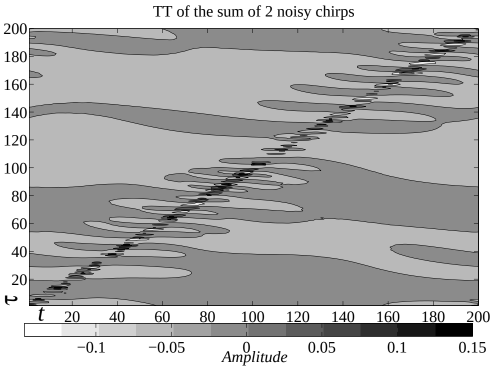
Figure 1: Two successive chirps, (a) without noise; (b) with noise.

We then compute the entire TT-transform of the two signals. As can be seen in Fig. 2 and as mentioned in the previous section, most of the information is located within a diagonal band and even on the diagonal.

We thus plot in Fig. 3 only the diagonal part, using the two methods: either taking the diagonal terms of the full TT-transform (2) or the direct formula of the diagonal



(a) The TT-transform of the two chirps shown in Fig.1(a).



(b) The TT-transform of the two noisy chirps shown in Fig. 1(b).

Figure 2: The TT-transform of chirps. It can be seen how the energy is concentrated on a diagonal band.

TT-transform (3). We first check that there is a perfect correspondence between both methods, the difference between both methods being plotted in Fig. 3b.

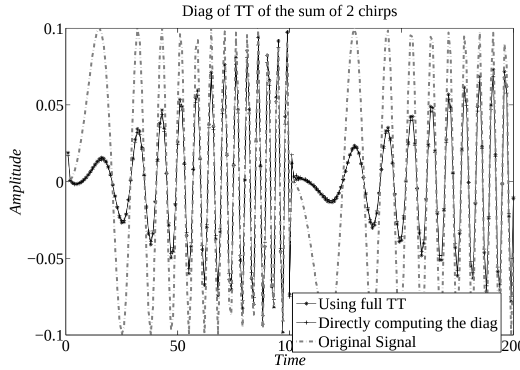
In Fig. 3a, the normalised original signal is also plotted together with its TT. It is clear from this figure that high frequencies are emphasised with respect to low ones. These characteristics will also be shown in the next example.

The second example is a sum of sines, Fig. 4. The top plot represents the sum of the sines while the bottom plot show the diagonal terms of its TT.

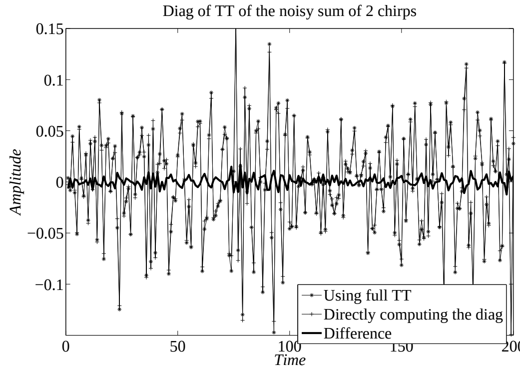
The FFT of the TT-transform has been normalised to facilitate the comparison with the FFT of the original signal, Fig. 5. In this plot, the TT-transform clearly emphasises high frequencies at the cost of low frequencies.

V. Conclusion

In this presentation, we show that computing the diagonal elements of the TT-transform of a signal is equivalent to frequency filtering it; the equivalent filter gives more emphasis to high frequencies with respect to low ones. As well as allowing a clear interpretation of the meaning of the diagonal of the TT-transform, this work thus gives a much simpler and more direct way to compute it.



(a) Diagonal of the TT of the two chirps of Fig. 1(a).



(b) Diagonal of the TT of the two noisy chirps of Fig. 1(b).

Figure 3: Comparing the two ways of computing the diagonal terms of the TT-transform. As the two ways nearly match, it is difficult to distinguish one from the other. For clarity, in Fig. 3b, the difference between the two techniques has also been plotted. In Fig. 3a, the original chirp is also plotted to illustrate the fact that the TT-transform emphasises high frequencies.

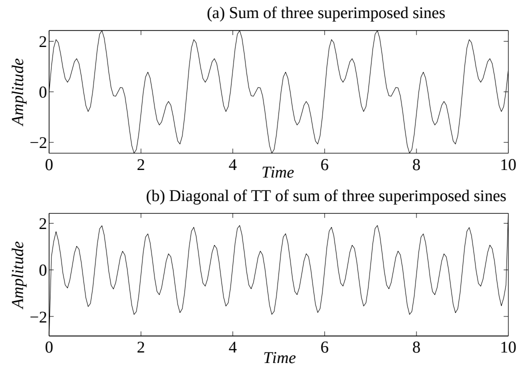


Figure 4: (a) Three sine functions of different frequency and, (b), the diagonal of its TT-transform.

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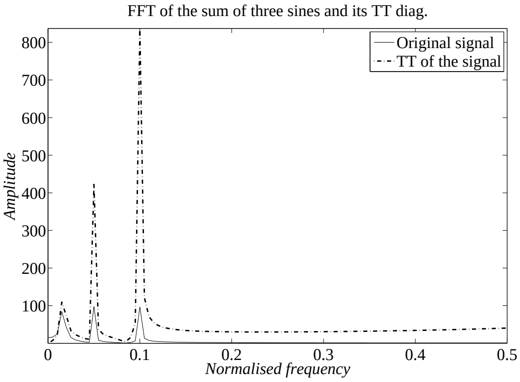

Figure 5: The FFT of the time series of Fig. 4 and of its TT diagonal. This plot clearly illustrates the kind of filter that the TT-transform performs, emphasising high frequencies.

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